1) a. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-3} = \frac{2t}{3(t+1)(t-1)}$.

Horizontal: $\frac{dy}{dx} = 0$ at $t=0 \to \boxed{(x,y) = (0,0)}$.

vertical: $\left|\frac{dy}{dx}\right| \to \infty$ at $t = \pm 1 \to$ $(x,y) = (-2,1)$, $t=1$. $(x,y) = (2,1)$, $t=-1$.

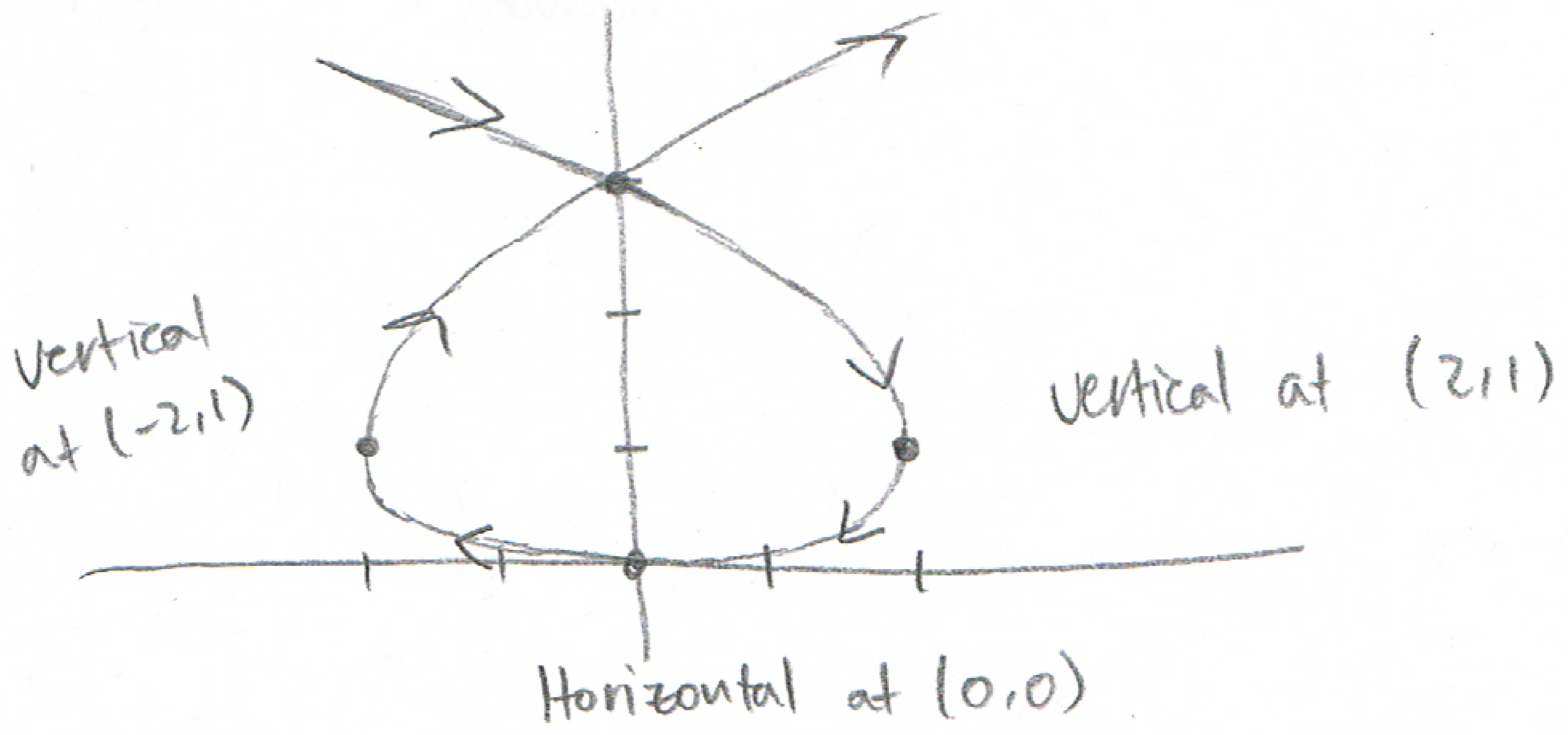
b. x-intercepts (when $y=0$): Only $t=0$ (at $(0,0)$)

y-intercepts (when $x=0$): $t^3-3t=0 \to t = 0, \pm\sqrt{3}$

These corespond to $(0,0)$ and $(0,3)$.

$\lim_{t\to\infty} c(t) \to (+\infty, +\infty)$ and $\lim_{t\to -\infty} c(t) \to (-\infty, +\infty)$.

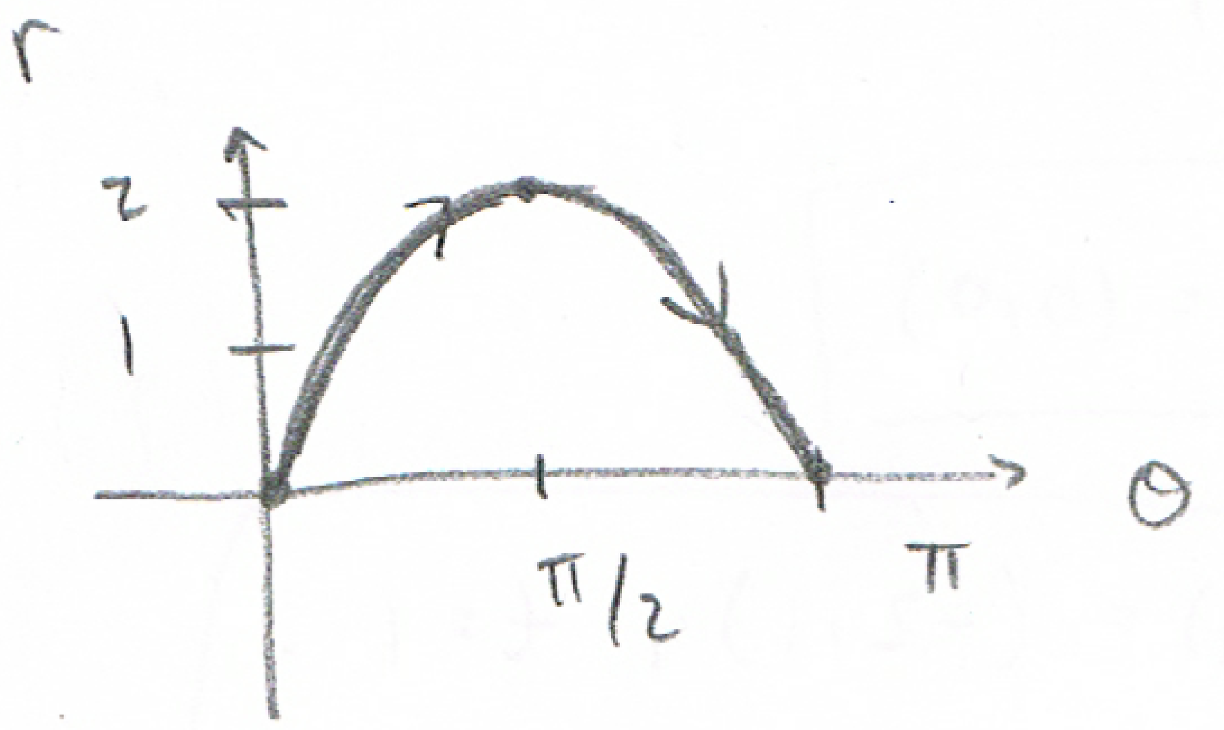
Graph:

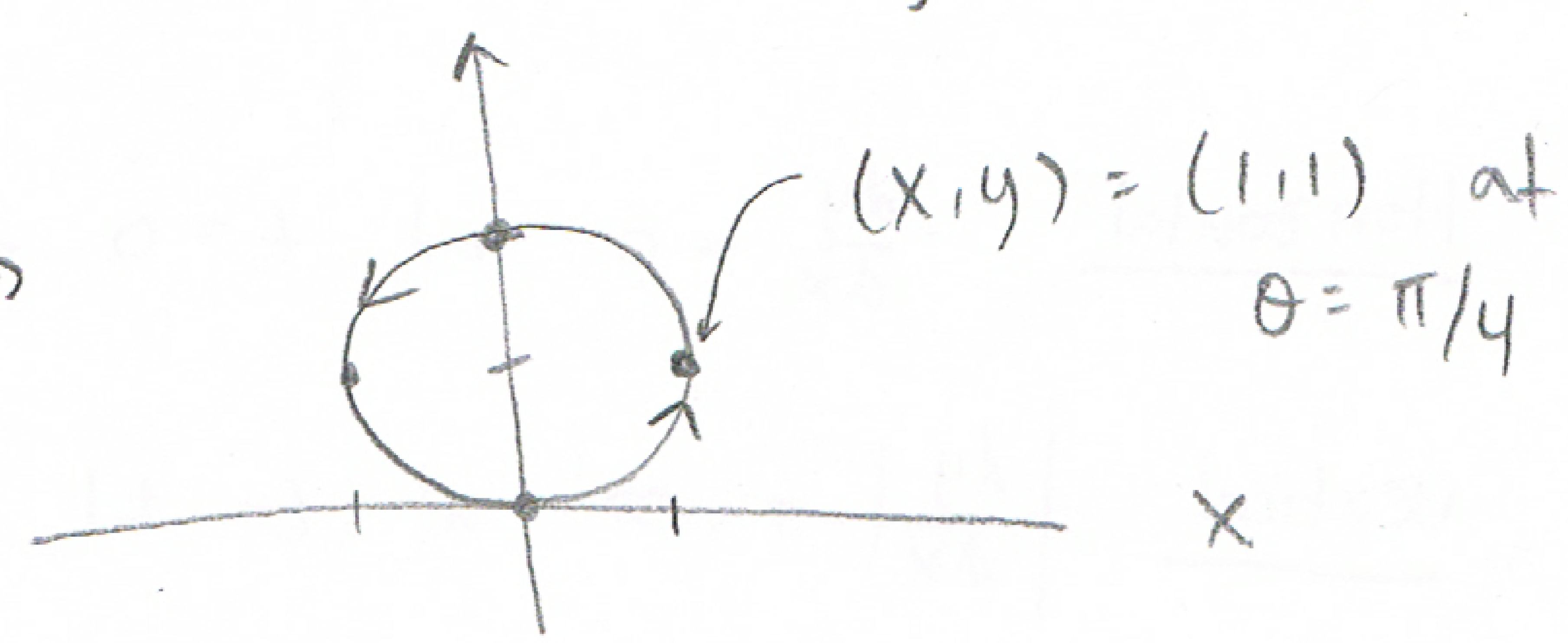


2) $r = 2\sin\theta$ on $0 \le \theta \le \pi$ is

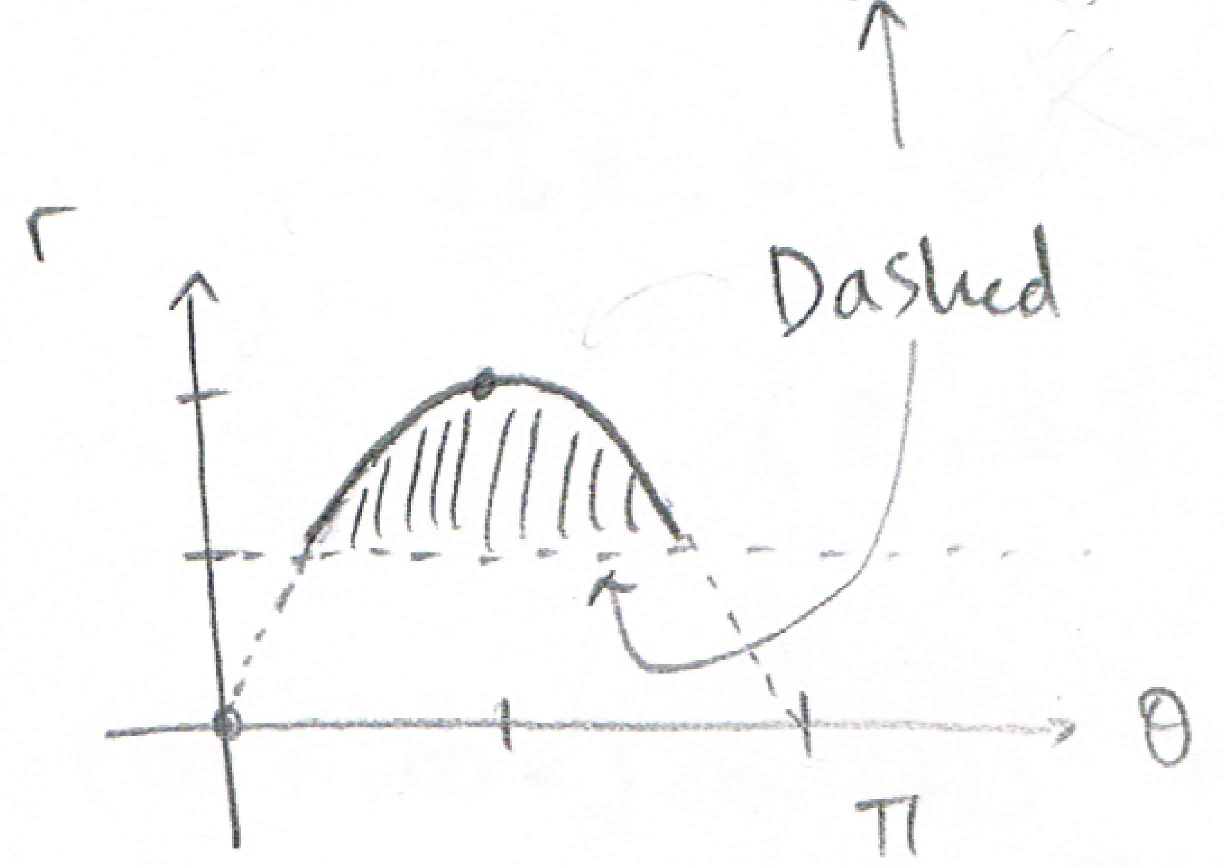
$x = r\cos\theta = 2\sin\theta\cos\theta$

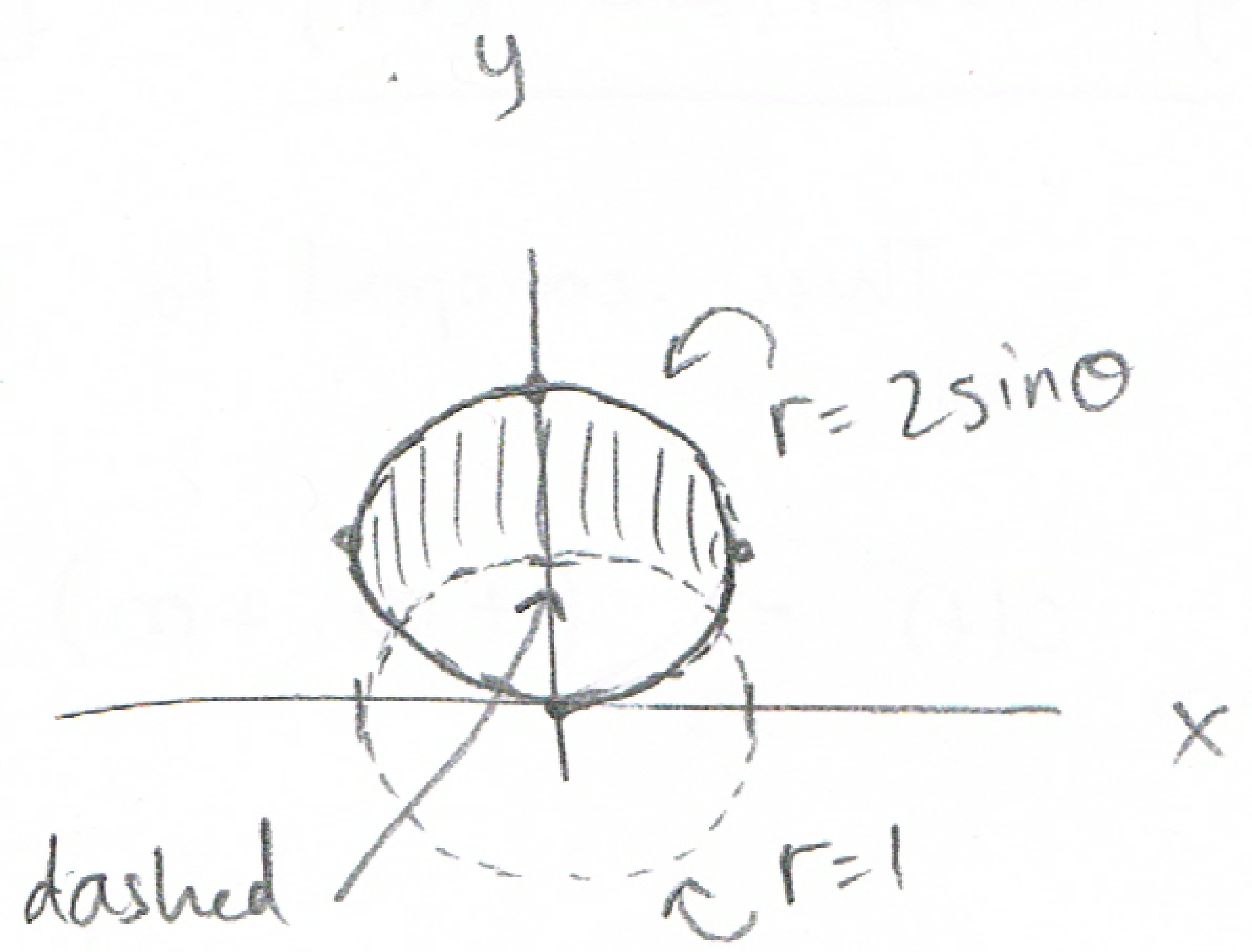
$y = r\sin\theta = 2\sin^2\theta$





3) If now $1 < r \le 2\sin\theta$ and $0 \le \theta \le \pi$,





4) a. $A=(1,1,-1)$ is not on $L=\langle t, 2t+1, 2-t\rangle$.

For x-component : $t=1$ only
For y-component : $t=0$ only.
} Not the same time! Thus, can't be on the line.

Eqn of line thru A parallel to L:

parallel $\Rightarrow$ same direction, $\langle a,b,c\rangle = \langle 1,2,-1\rangle$.

Use point $A=(1,1,-1)$ $\rightarrow$ $\boxed{\begin{aligned} x&=1+t\\ y&=1+2t\\ z&=-1-t\end{aligned}}$

(Better one)

b. $\boxed{\text{Soln } 1}$ Note: $\dfrac{\sqrt{66}}{3}=\dfrac{\sqrt{44}}{\sqrt{6}}$

The vector from A to line L must be $\perp$ to the direction at the shortest distance.

let $\vec{v}=\overrightarrow{AL}=\langle t-1, 2t+\cancel{1}-\cancel{1}, 2-t+1\rangle$

Then $\vec{v}\cdot$ direction of $L=0$

ie. $\vec{v}\cdot\langle 1,2,-1\rangle$

$=(t-1)+4t-(3-t)=0$

ie. $6t-4=0$, $t=2/3$

Then distance $=|\vec{v} \text{ at } t=2/3|$

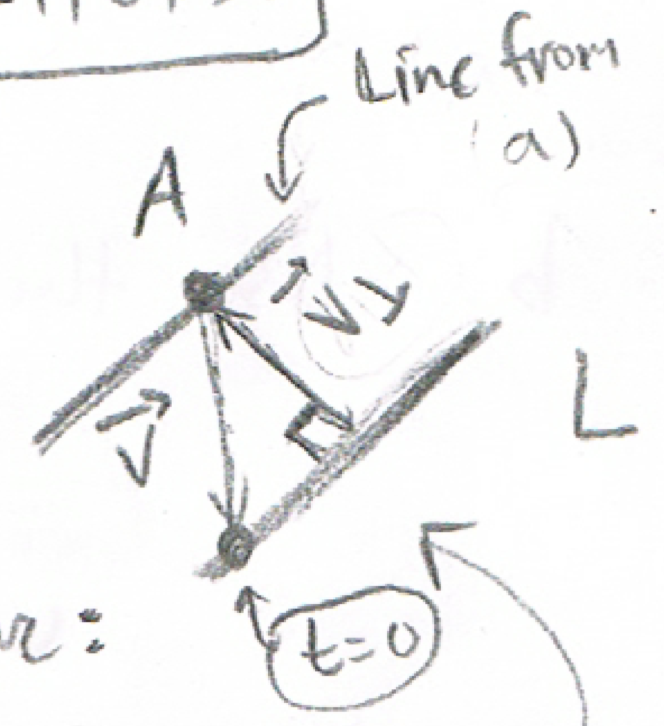
$=\left|\left\langle -\frac{1}{3}, \frac{4}{3}, \frac{7}{3}\right\rangle\right| = \dfrac{\sqrt{66}}{3}$

Let $\vec{v}=$ A to $L(t=0)$ $\langle 0,1,2\rangle$

So $\boxed{\vec{v}=\langle -1,0,3\rangle}$

(start)

Soln 2



Geometrically we have:

The distance is the $\perp$ part of $\vec{v}$, ie. length of $\boxed{\vec{v}_\perp=\vec{v}-\text{proj}_{\vec{n}}\vec{v}}$

where $\boxed{\vec{n}=\langle 1,2,-1\rangle}$ Line L's direction.

[We can compute $\vec{v}_\perp$ and $\boxed{\text{distance}=|\vec{v}_\perp|}$]

Alternatively: Recall cross product doesn't feel parallel parts, so

$|\vec{v}_\perp\times\vec{n}|=|\vec{v}_\perp||\vec{n}|\sin 90^\circ$, and

from quiz 2; $|\vec{v}_\perp\times\vec{n}|=|\vec{v}\times\vec{n}|$.

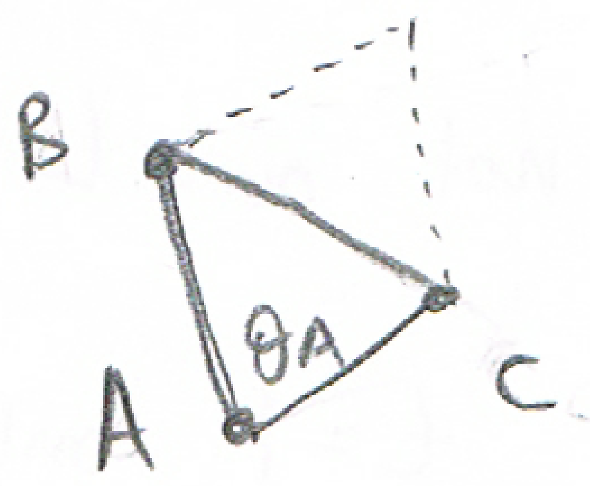
Thus, $|\vec{v}_\perp\times\vec{n}|=|\vec{v}\times\vec{n}|=|\vec{v}_\perp||\vec{n}|$.

Rearrange/solve: $\boxed{\text{dist}=|\vec{v}_\perp|=\dfrac{|\vec{v}\times\vec{n}|}{|\vec{n}|}=\dfrac{\sqrt{44}}{\sqrt{6}}}$

c. Plane $\perp$ to L implies L is along its normal. $A=(1,1,-1)$

Thus, $\vec{n}=\langle 1,2,-1\rangle$, the direction of L $\Rightarrow$ Eqn: $\boxed{(x-1)+2(y-1)-(z+1)=0}$

5) a. $A_T = \frac{1}{2} A_{\text{parallelogram}}$



$A_{\text{para}} = |\vec{AB} \times \vec{AC}|$

$= |\langle 1,0,1\rangle \times \langle -1,1,2\rangle| = \left| \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ -1 & 1 & 2 \end{pmatrix} \right|$

$= |-1\hat{i} - 3\hat{j} + \hat{k}| = \sqrt{11}$.

Thus $\boxed{A_T = \frac{\sqrt{11}}{2}}$

b. Use the normal as $\vec{AB} \times \vec{AC}$; use A as a pt on plane.

↳ $\boxed{-(x-1) - 3(y-1) + (z+1) = 0}$

(Reduces to: $-x - 3y + z = -5$)

c. Let this angle be θ_A.

↳ We then know $\vec{AB} \cdot \vec{AC} = |\vec{AB}|\,|\vec{AC}| \cos\theta_A$.

- $\vec{AB} \cdot \vec{AC} = -1 + 0 + 2 = 1$
- $|\vec{AB}| = \sqrt{2}$ and $|\vec{AC}| = \sqrt{6}$.

Thus, $\cos\theta_A = \frac{1}{\sqrt{2}\sqrt{6}}$, ie. $\boxed{\theta_A = \cos^{-1}\left(\frac{1}{\sqrt{12}}\right)}$.